More on linear regression analysis

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What we are going to learn

- Interaction effects

Birthweight data

- File: birthsmokers.txt
- N = 32 births
- Baby's birth weight is related mother's smoking habits during pregnancy
- Response (y): birth **weight** in grams of baby
- Potential predictor (x_1):
  - **smoking** status of mother (yes or no)
  - Potential predictor (x_2): length of **gestation** in weeks

Birthweight data

- The model:

$$y_i = b_0 + b_1 x_{i1} + b_2 x_{i2} + e_i$$

- y_i is the birth weight of baby *i* in grams
- x_{i1} is the length of gestation of baby *i* in weeks
- $x_{i2} = 1$, if baby *i*'s mother smoked and $x_{i2} = 0$, if not
- ε_i error terms follow a normal distribution with mean 0 and equal variance σ^2.

Birthweight data

- **The model:**

$$y_i = b_0 + b_1 x_{i1} + b_2 x_{i2} + e_i$$

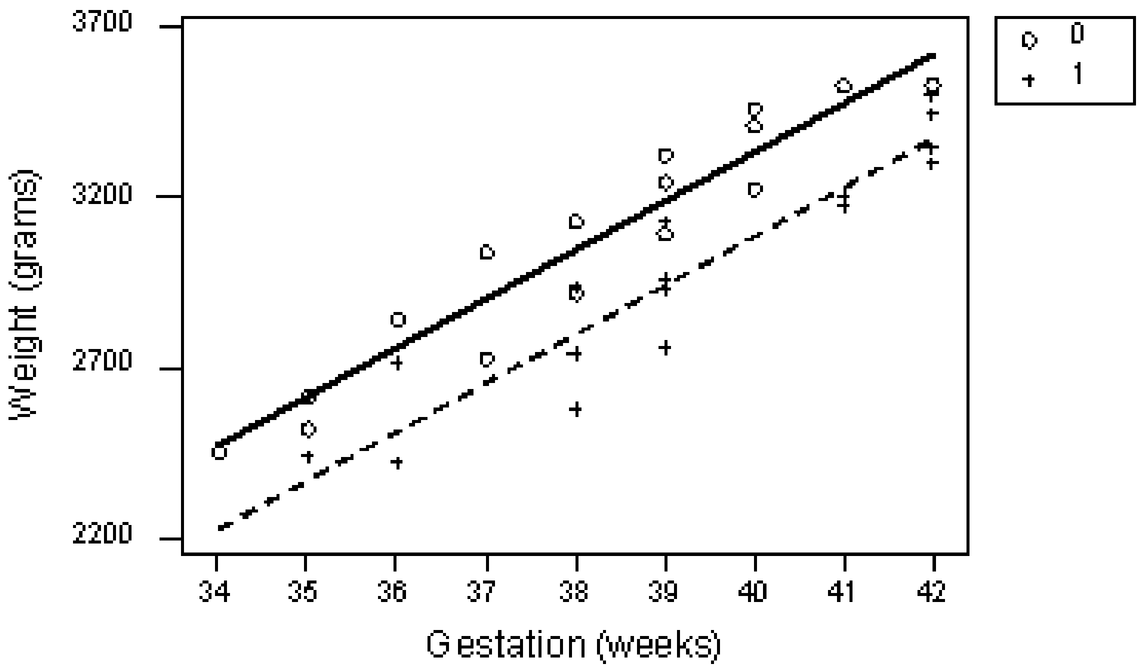
- **The rehression equation based on actual data**

Weight = -2390 + 143*Gest – 245*Smoking

Birthweight data

- **The rehression equation based on actual data**

Weight = -2390 + 143*Gest – 245*Smoking

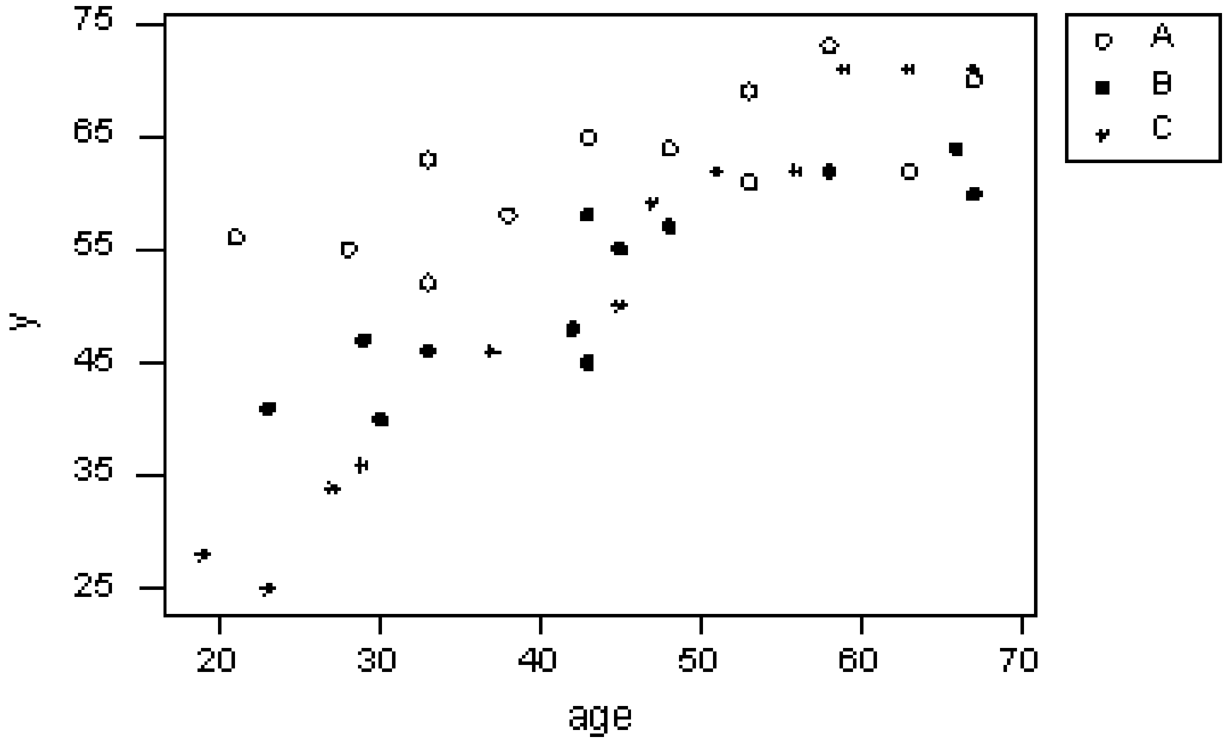


No interaction effects!

Interaction effects

- Depression.txt, 36 patients with depression
- Comparing the effectiveness of 3 treatments for severe depression (A, B, and C)
- y_i = measure of the effectiveness of the treatment for individual i
- x_{i1} = age (in years) of individual i
- x_{i2} = 0 if individual i received treatment A, 1 if treatment B, 2 if treatment C

Interaction effects



Interaction model

- **The "main effect" regression model**

$$y_i = b_0 + b_1 x_{i1} + b_2 x_{i2} + e_i$$

- **The "interaction effect" regression model**

$$y_i = b_0 + b_1 x_{i1} + b_2 x_{i2} + b_3 x_{i1} x_{i2} + e_i$$

R codes

```
dep =
read.table("/Users/tuannguyen/Documents/_Viet
nam2012/Can Tho /Datasets/depression.txt",
header=T)

attach(dep)

res = lm(y ~ age+TRT)

summary(res)
```

Results of main effect model

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   32.54335    3.58105   9.088 2.23e-10 ***
age            0.66446    0.06978   9.522 7.42e-11 ***
TRTB          -9.80758    2.46471  -3.979 0.000371 ***
TRTC         -10.25276    2.46542  -4.159 0.000224 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
' ' 1

Residual standard error: 6.035 on 32 degrees of freedom
Multiple R-squared: 0.784,    Adjusted R-squared: 0.7637
F-statistic: 38.71 on 3 and 32 DF,  p-value: 9.287e-11
```

Interpretation of the main model

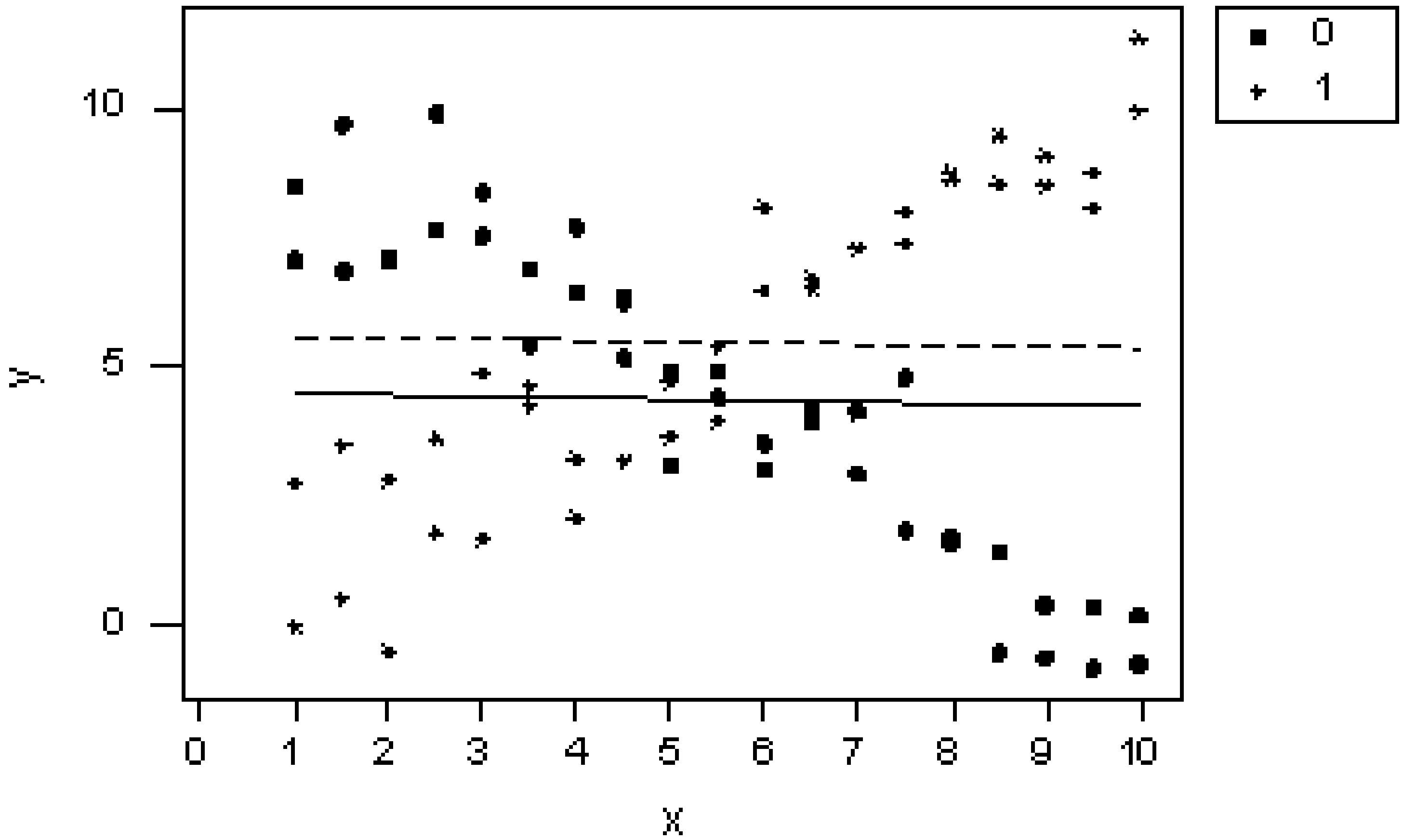
```
(Intercept)  32.54335    3.58105   9.088 2.23e-10 ***
age           0.66446    0.06978   9.522 7.42e-11 ***
TRTB         -9.80758    2.46471  -3.979 0.000371 ***
TRTC        -10.25276    2.46542  -4.159 0.000224 ***
```

Treatment A: y = 32.54 + 0.66*age

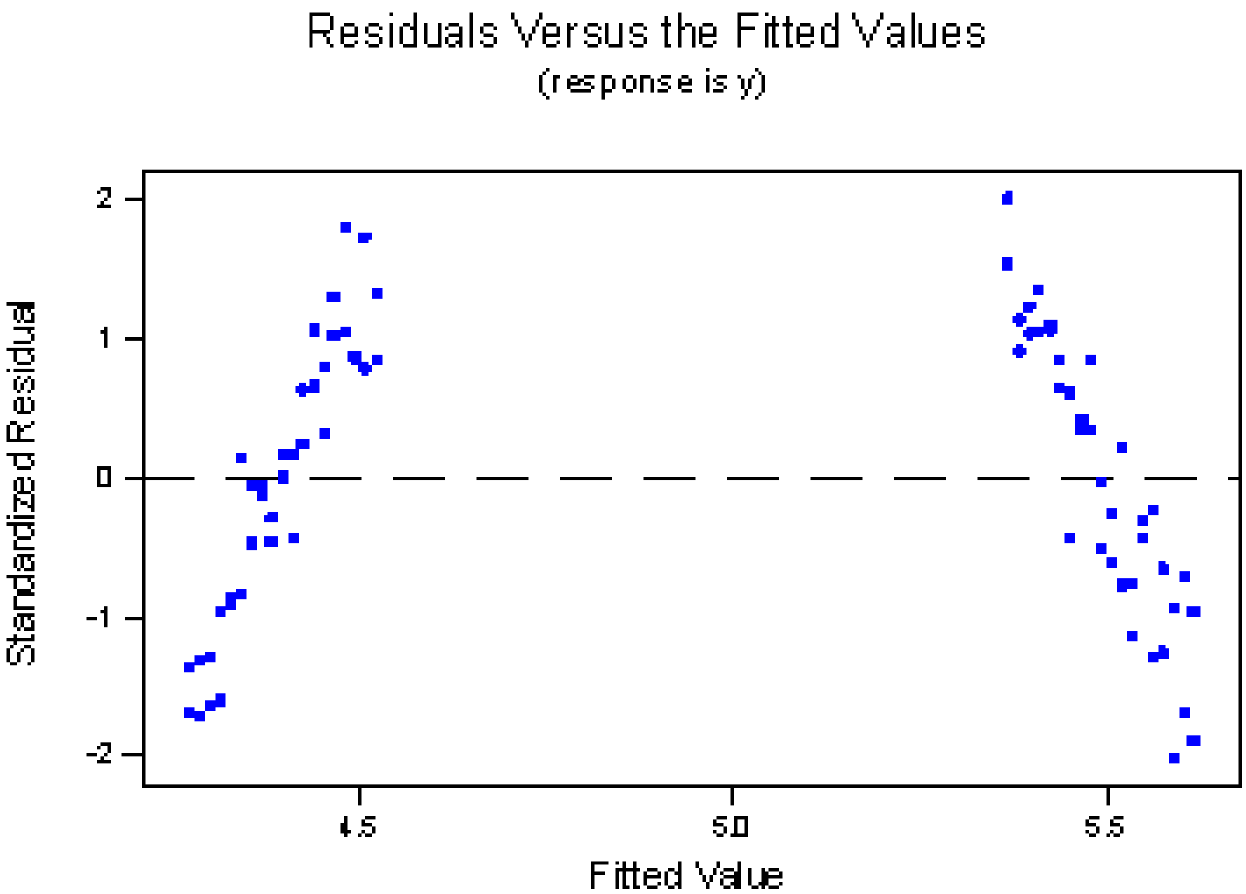
Treatment B: y = 32.54 + 0.66*age – 9.81
= 22.73 + 0.66*age

Treatment C: y = 32.54 + 0.66*age – 10.25
= 22.29 + 0.66*age

The model in graphical format



Residual analysis of "main effect" model



R codes for interaction model

```
dep =
read.table("/Users/tuannguyen/Documents/_Vietnam2012/Can Tho /Datasets/depression.txt",
header=T)

attach(dep)

int = lm(y ~ age+TRT+age:TRT)

summary(int)
```

Results of interaction effect model

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     47.51559    3.82523  12.422 2.34e-13 ***
age              0.33051    0.08149   4.056 0.000328 ***
TRTB           -18.59739    5.41573  -3.434 0.001759 **
TRTC           -41.30421    5.08453  -8.124 4.56e-09 ***
age:TRTB         0.19318    0.11660   1.657 0.108001
age:TRTC         0.70288    0.10896   6.451 3.98e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
' ' 1

Residual standard error: 3.925 on 30 degrees of freedom
Multiple R-squared: 0.9143,   Adjusted R-squared: 0.9001
F-statistic: 64.04 on 5 and 30 DF,  p-value: 4.264e-15
```

“Interpretation” of model

```
Estimate Std. Error t value Pr(>|t|)
(Intercept)  47.51559    3.82523  12.422 2.34e-13 ***
age           0.33051    0.08149   4.056 0.000328 ***
TRTB        -18.59739    5.41573  -3.434 0.001759 **
TRTC        -41.30421    5.08453  -8.124 4.56e-09 ***
age:TRTB      0.19318    0.11660   1.657 0.108001
age:TRTC      0.70288    0.10896   6.451 3.98e-07 ***
```

The model is:

If treatment=A, then y = 47.51 + 0.33*age

If treatment=B, then y = 47.51 + 0.33*age – 18.60 + 0.19*age

= 28.91 + 0.52*age

If treatment=C, then y = 47.51 + 0.33*age – 41.30 + 0.70*age

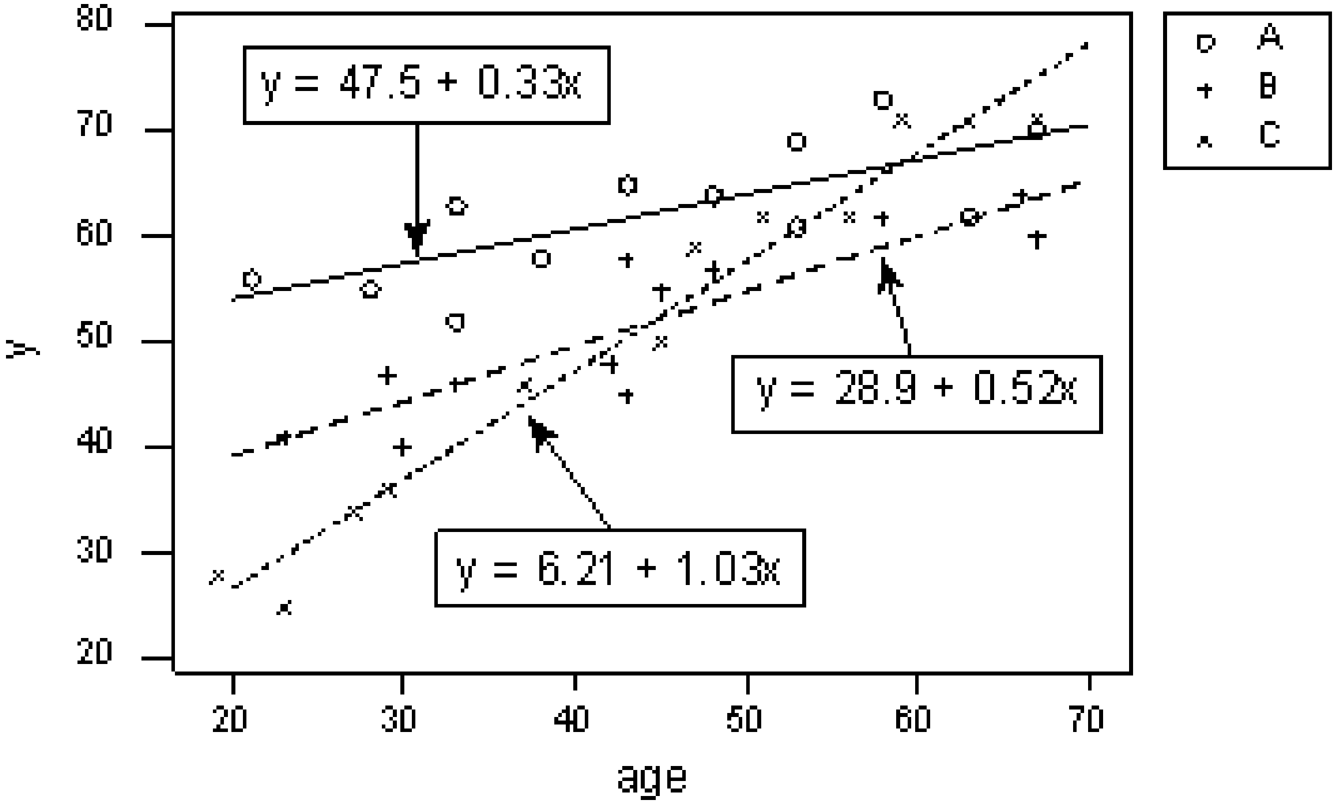
= 6.21 + 1.03*age

"Interpretation" of model

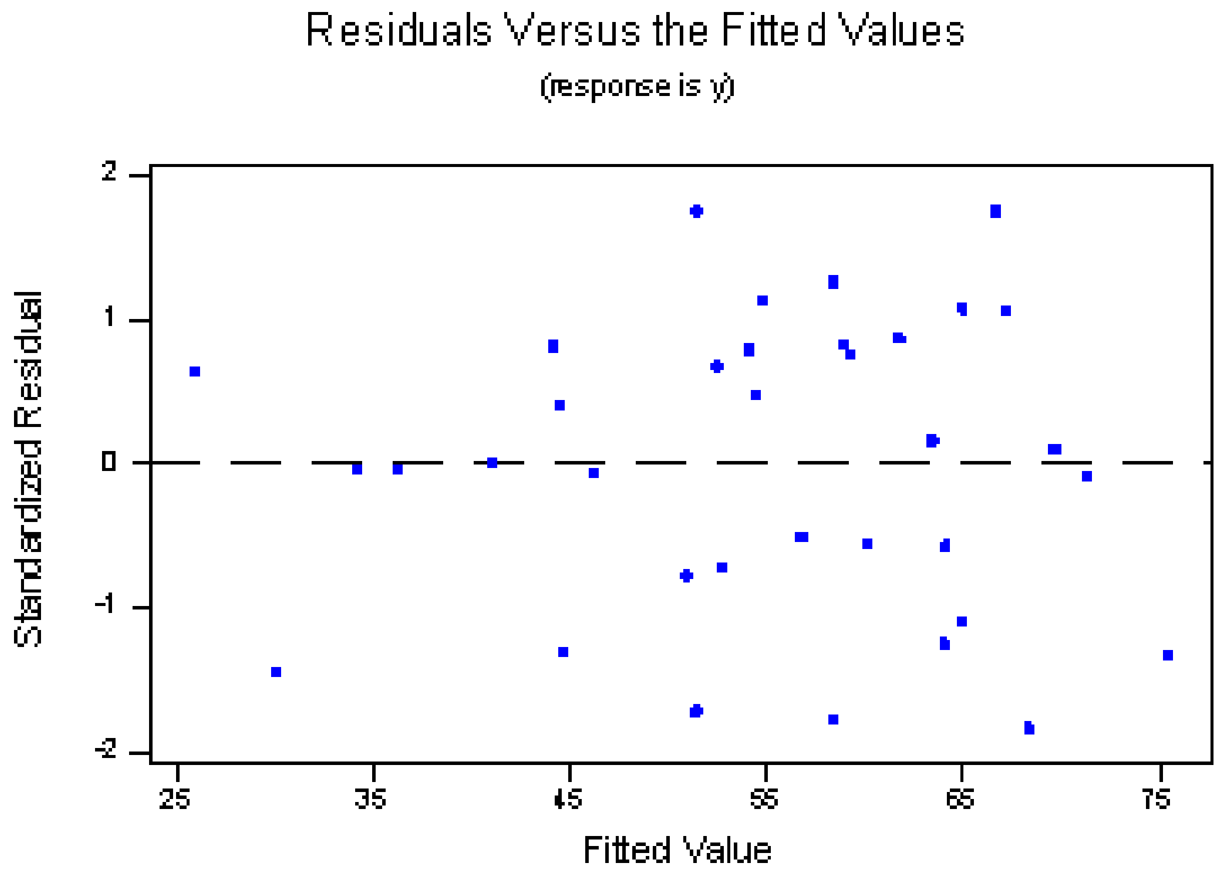
$y_A = 47.51 + 0.33 * age$

$y_B = 28.91 + 0.52 * age$

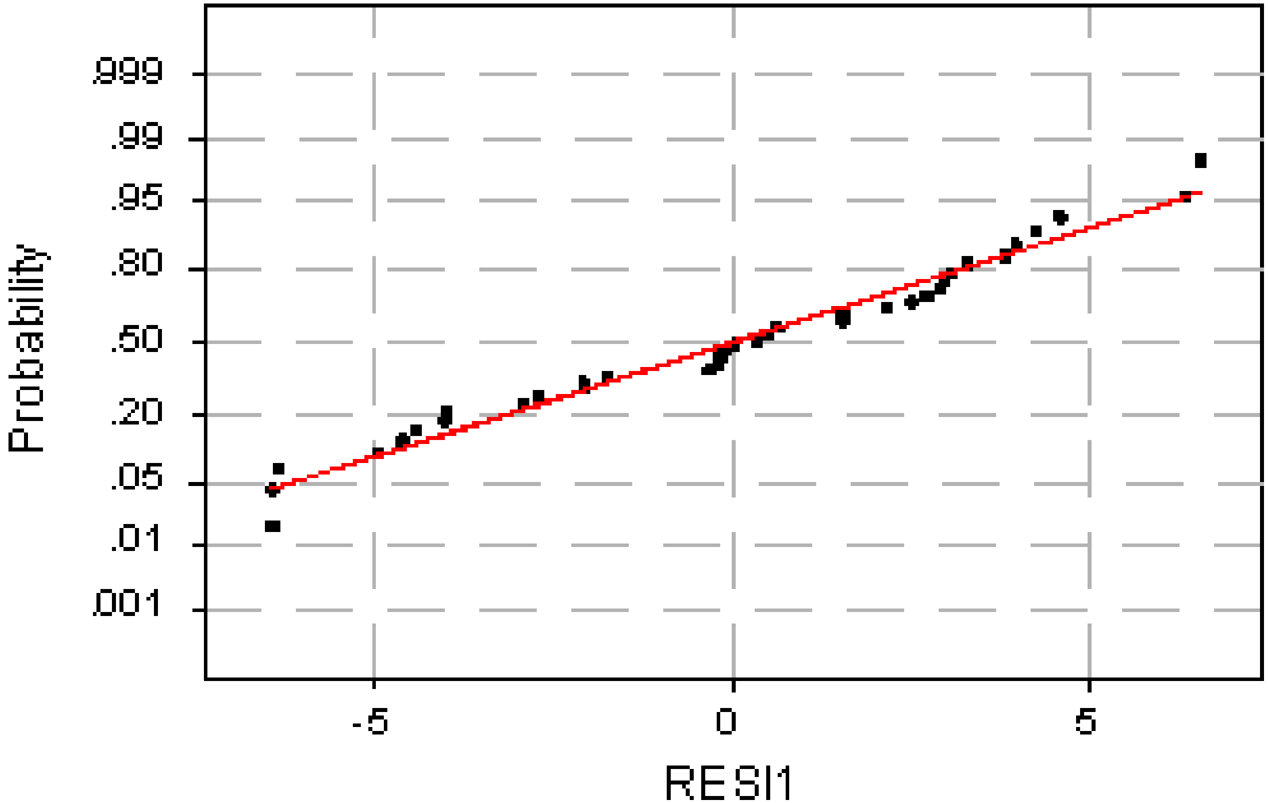
$y_C = 6.21 + 1.03 * age$



Residual analysis



Normal plot



Average: -0.0000000
StDev: 3.63376
N: 36

W-test for Normality
R: 0.9866
P-Value (approx): > 0.1000